whereby a few pounds of any propellant may be used to investigate oscillatory combustion in the frequency range of large, solid propellant boosters designed to employ several tons of propellant. Such a tool then may materially aid the development of large systems by providing propellant evaluation without expensive full-scale motor firings.

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Deformations and Stresses in Axially Loaded and Heated Cylindrical Shells

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THE authors have recently conducted some numerical studies to assess the effects of axial thrust on the bending stresses and deflections in cylindrical booster tanks subjected to longitudinal temperature variations. The calculations were based on the following displacement equation of equilibrium:

$$D\frac{d^4w}{dx^2} - N_x\frac{d^2w}{dx^2} + \frac{Ehw}{R^2} = \frac{\nu N_x - N_t}{R} \qquad (1)$$

where

$N_t = \int_{-h/2}^{h/2} \alpha E T(x) dz$

x = axial coordinate measured from a cylinder edge

z = thickness coordinate, positive inward

h,R = thickness and radius of cylinder, respectively

and the remaining notation is the same as in Ref. 1. The second term on the left side of Eq. (1) represents the additional transverse loading due to N_x which is included in the classical buckling equation.

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With the assumption that the edges be free of restraint, the boundary conditions are given by

$$\left.\begin{aligned} d^2w/dx^2 &= 0 \\ (d^3w/dx^3) - (N_x/D)(dw/dx) &= 0 \end{aligned}\right\} \text{ at } x = 0,\ x = L \qquad (2)$$

Assume that N_t is analytic and thereby expressible in the form of a power series. The nondimensional solution of (1) is

$$\bar{w} = C_1 \cosh\gamma\xi \cos\delta\xi + C_2 \sinh\gamma\xi \cos\delta\xi + C_3 \cosh\gamma\xi \sin\delta\xi + C_4 \sinh\gamma\xi \sin\delta\xi + \sum_{k=0}^{\infty} \frac{B_k}{\lambda^2} \sum_{j=0,1,\ldots}^{[k/2]} A_{k-2j}\, \xi^{k-2j} \qquad (3)$$

where

$$\bar{N}_t = -\sum_{k=0}^{\infty} B_k\xi^k = \frac{12(1-\nu^2)}{ER}\left(\frac{L}{h}\right)^4 N_t$$

$$\xi = \frac{x}{L}$$

$$\lambda^2 = 12(1-\nu^2)\left(\frac{L}{R}\right)^4\left(\frac{R}{h}\right)^2$$

$$\beta = \frac{-N_x R[3(1-\nu^2)]^{1/2}}{Eh^2}$$

$$w = \frac{w}{h} - \frac{\nu R}{Eh^2} N_x$$

$$\gamma = \left[\frac{\lambda}{2}(1-\beta)\right]^{1/2} \qquad \delta = \left[\frac{\lambda}{2}(1+\beta)\right]^{1/2}$$

and C_i are arbitrary constants. The quantities A_{k-2j} are given by[2]

$$A_{k-2j} = \frac{(-1)^j k!}{(k-2j)!}\left[\left(\frac{2\beta}{\lambda}\right)^j + \sum_{r=2}^{[(j/2)+1]} \frac{(-1)^{r-1}(j-r+1)(j-r)\ldots(j+3-2r)}{(r-1)!\,4^{r-1}} \times \left(\frac{2\beta}{\lambda}\right)^{j-2(r-1)}\left(\frac{4}{\lambda^2}\right)^{r-1}\right]$$

$[N]$ is the largest integer less than or equal to N.

The dimensionless bending stresses in the longitudinal and hoop directions are calculated from

$$\bar{\sigma}_{xb} = -\left(\frac{3}{1-\nu^2}\right)^{1/2}\frac{1}{\lambda}\frac{d^2\bar{w}}{d\xi^2} = \frac{\bar{\sigma}_{\phi b}}{\nu} \qquad (4)$$

where

$$(\bar{\sigma}_{xb}, \bar{\sigma}_{\phi b}) = (R/Eh)(\sigma_{xb}, \sigma_{\phi b})$$

These must be superposed on the direct stresses

$$\bar{\sigma}_{xd} = -\frac{\beta}{[3(1-\nu^2)]^{1/2}} \qquad \bar{\sigma}_{\phi d} = -\left(\bar{w} + \frac{N_t}{\lambda^2}\right) \qquad (5)$$

where

$$(\bar{\sigma}_{xd}, \bar{\sigma}_{\phi d}) = (R/Eh)(\sigma_{xd}, \sigma_{\phi d})$$

Deflections for a temperature of the form $T = T_0\xi^2$ are shown in Fig. 1. Axially loaded to unloaded bending stress ratios for this temperature distribution and, in addition, for $T = T_0\xi^3$ are given in Fig. 2. All results are for a cylinder geometry parameter of $\lambda = 600$ (corresponding, for example, to $R/h \approx 1000$ and $L/R \approx 0.40$). It is seen that the deflections are rather insensitive to end load, whereas the bending stresses due to end load may be many times that predicted by elementary superposition. For example, if $T/T_0 = \xi^2$, this ratio is 7.5 for $\beta = 0.2$ (0.4 of the buckling load),[3] showing the adverse effect of the end loads.

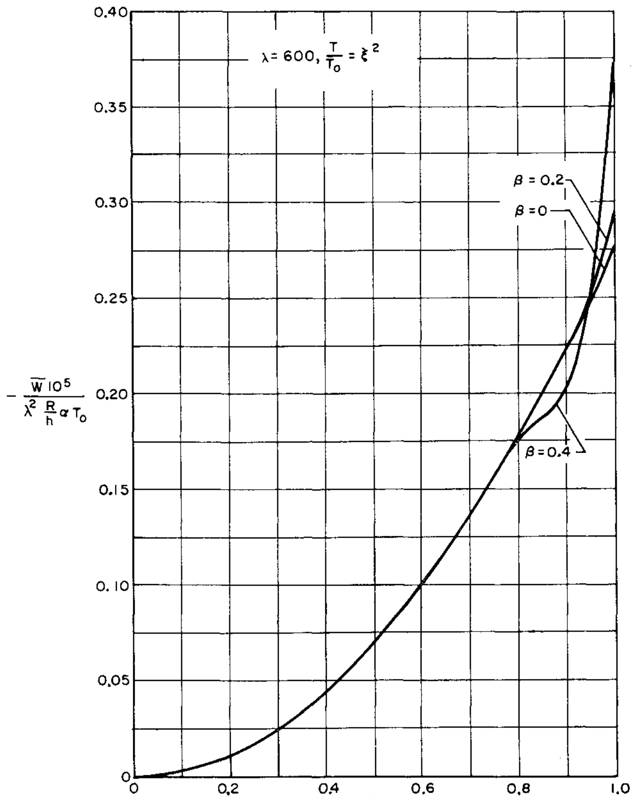

Fig. 1 Deflection vs axial position with end load as parameter.

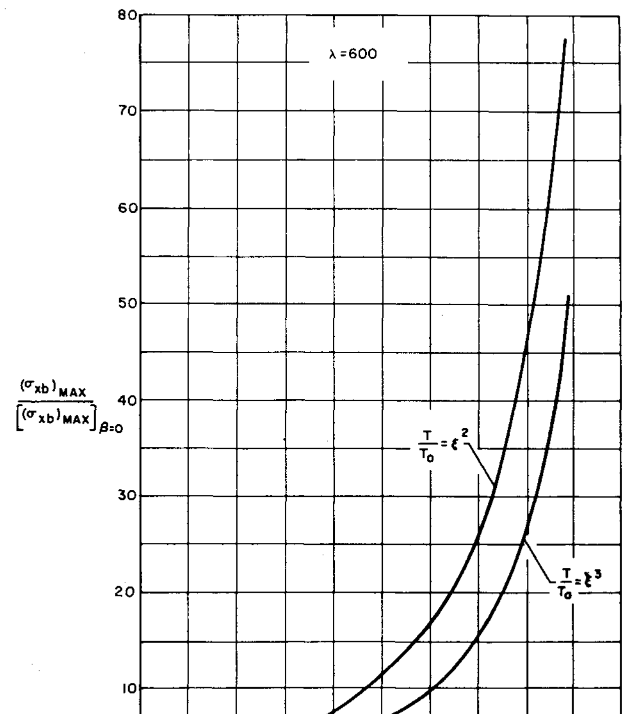

Fig. 2 Ratio of nondimensional bending stresses for parabolic and cubic temperature distributions.

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Radar Determination of Lunar Surface Dielectric Properties

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THEORETICAL reflection coefficient of an idealized spherical lunar surface is used to discuss ways and means to obtain its dielectric constants. Various estimates of these constants are compared, and then validity is checked. On the basis of this and other lunar theories, it is concluded that the presence or absence of a dust layer on the moon cannot be definitely established with available lunar echo data, although a fairly strong indication can be obtained by multifrequency experiments, from near the lunar surface, using lunar space probes.

The average distance of the moon from the earth is approximately 221 lunar radii, and therefore a spherical wave front can be safely approximated by a plane wave front at the surface of the moon. Moreover, it is well established[1,2] that nearly 50% of the lunar echo power is returned from the central area lying within a circle of radius 105 miles, or approximately $\frac{1}{10}$ of the radius of the moon. This leads to the approximation that the angle of incidence, measured from the outward surface normal to the propagation vector on the lunar surface is nearly zero. Assuming the surface permeability as unity, the Fresnel reflection factor R for a plane wave incident on a spherical surface is[6]

$$R = D_s^2 \left[\frac{(e_c)^{1/2} - 1}{(e_c)^{1/2} + 1} \right]^2 \tag{1}$$

where

D_s = divergence factor, in order to account for the spherical shape of the reflecting surface $\approx a/h$ (for vertical incidence)
a = radius of the sphere, (moon)
h = distance of the receiver from the moon surface
e_c = $(e/e_0) - j(s/we_0)$, relative complex dielectric constant of the central portion of the lunar surface
e_0 = free space dielectric constant
s = conductivity of lunar surface

The calculation of R/D_s^2 for a single frequency of transmission would result in an absolute value of e_c or

$$|e_c|^2 = (e/e_0)^2 + (s/we_0)^2 \tag{2}$$

It is possible to calculate the constants e and s for the lunar surface if a controlled lunar echo experiment is performed using transmission at several frequencies, which cover a range of about two octaves. The foregoing statements assume e and s to be approximately independent of frequency, in the absence of which such calculations become very complicated. It is pertinent to state that the dielectric properties so calculated would give only effective values for the

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